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ORIGINAL ARTICLE

# Height datum unification between Shenzhen and Hong Kong using the solution of the linearized fixed-gravimetric boundary value problem

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**Abstract** The paper proposes a new algorithm to unify height datums in different regions, which is based on the solution of the linearized fixed-gravimetric boundary value problem. Compared with traditional methods, this method uses GPS ellipsoidal height and gravity disturbances on the surface of the earth to obtain a quasigeoid, which is not related to any local vertical datums. As an example, we calculate the height datum difference between Shenzhen and Hong Kong by applying this new method. The result shows that the height difference obtained by this new method is consistent with the ground leveling result to a few centimeters.

**Keywords** Height datum · Height system unification · Geodetic boundary value problem (GBVP) · Quasigeoid

## 1 Introduction

In geodesy, there are mainly two types of height systems: the geometrical height based on ellipsoid and the physical height based on gravity-defined surfaces (i.e. orthometic and normal heights). Along with the establishment of highly precise International Terrestrial Reference Frame (ITRF) and the development of satellite navigation systems, we can determine the ellipsoidal height at any location on the earth to the accuracy of centimeter level. On the other hand, the physical heights are traditionally referenced to local datums [i.e., local mean sea level (MSL)]. For a long time, MSL has been regarded as the reference surface for height, which is generally determined by a certain form of average on sea level observation data by one or several long-term tide gauges. Because of sea surface topography (SST), the vertical datum differences in different regions can exceed 2 m (Jiao et al. 2002). As the basis of height, a regional or global unified vertical datum is of great importance to construct regional or global geospatial information systems, and to study SST at different tide gauges. The definition of a global height reference system is based on a mean sea surface, gravity field parameters, and a three-dimensional terrestrial reference frame (Ihde and Sanchez 2005). Global height datum unification has been an important research in geodesy and studied by many research groups, such as Sanso and Usai (1995), Featherstone (1998, 2000), Rapp (1995), Colombo (1980), Grafarend and Ardalan (1997, 1999, 2002), Ardalan and Safari (2005), Xu and Rummel (1991), Nahavandchi and Sjoberg (1998), Pan and Sjoberg (1998), Rummel and Ilk (1995), Rummel (2000), Heck and Rummel (1990), Jekeli (2000, 2003), Hipkin (2002), Poutanen (1999), and Sacerdote and Sanso (2001).

The key issue of height datum unification is to determine the potential differences among different height datums (Rummel 2000). Different approaches have been studied for this purpose. Geodetic leveling is a direct measurement of potential differences by combining spirit leveling and gravity data. The accuracy of this method is very high. However, it cannot connect two height datums separated by oceans. Ocean leveling (Ekman 1999) is another method to establish the potential difference between two points across ocean, and

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generally includes steric, dynamic, and altimetric leveling. However, the accuracy of ocean leveling is relatively low, due to the sparseness of ocean data, the time variability of the ocean, geostrophic assumption, poor reliability of radar altimetry close to the coast, and lack of high precision tide models (Rummel 2000). Burša et al. (1999, 2001, 2004) propose a method to use a large area covered with GPS/leveling data to determine the mean bias between a local geoid and the global geoid, which is defined by a global gravity potential model, i.e. EGM96 (Lemoine et al. 1998). Global gravity potential models usually reflect the middle and low frequency components in the gravity field, and the precision of existing gravity models is still at the decimeter level. Therefore, GPS/leveling data from a reasonable large area is needed to smoothen the high frequency gravity components. The solutions of the geodetic boundary value problems (GBVP) that determine the potential difference between two areas are also applied for height unification (i.e. Colombo 1980; Heck and Rummel 1990; Rummel and Teunissen 1988; Sanso and Usai 1995). As gravity data are strongly correlative with terrain, gravity residuals (after removal of the global gravity model) can be used to represent the high frequency components of the gravity field. However, these methods require the use local heights (which is related to a local datum) to calculate the gravity anomaly. Based on potential theory, using a solution of the fixed-free two-boundary-value problem (Ardalan and Grafarend 2004), a new method of global height datum unification has been put forward by Ardalan and Safari (2005).

Ideally, if we can find a surface, which is irrelevant to any local height datums, we can compare local (quasi)geoids to this common surface to determine the potential differences among different regions. The linearized fixed-gravimetric BVP (Sacerdote and Sanso 1989; Fei and Sideris 2001; Li et al. 2003; Yu et al. 2003) provides a solution. Based on the solution of the linearized fixed-gravimetric BVP, we can use GPS positions and gravity disturbances to determine an accurate quasigeoid. With the establishment of the International GNSS Services (IGS), GPS positions can be easily obtained in a global reference frame (i.e. ITRF). Gravity measurements in different regions can be tied to an International Absolute Gravity Basestation Network (IAGBN, http://www.gfy.ku.dk/~iag/Travaux_99/comm3.htm). As long as we apply the same fundamental "constants" of geodesy (i.e. the ellipsoidal parameters, the geocentric gravitational constant GM, and the nominal mean angular velocity of the earth rotation), the quasigeoid obtained through the linearized fixed-gravimetric BVP is independent of any local datums.

In this paper, we will briefly introduce the linearized fixed-gravimetric BVP, which belongs to the second GBVP, and its asymptotic solution (Li et al. 2003, 2005). Then, the principle of the determination of gravity potential difference between the two local height datums based on the solution of the linearized fixed-gravimetric BVP is given. As an example, we use GPS, gravity and leveling data in Shenzhen and Hong Kong, China, to determine the difference between the Shenzhen and the Hong Kong height datums. Leveling data are used to validate the method.

## 2 The linearized fixed-gravimetric BVP and its application for height datum unification

The linearized fixed-gravimetric BVP [also known as GPS/Gravity BVP in Li et al. (2003, 2005)] can be defined as the one with the earth surface as boundary and gravity disturbances on it as boundary value, to determine (quasi) geoid and external gravity field.

Suppose that no mass are outside the earth surface, the linearized fixed-gravimetric BVP can be mathematically described as

$$\begin{cases} \Delta T = 0 & \text{on and outside the earth surface } S \\ \frac{\partial T}{\partial h}\big|_S = -\delta g & \text{on the earth surface } S \\ T = o(1/r) & r \to \infty \end{cases} \tag{1}$$

where $\Delta$ is the Laplace operator, $T$ is the disturbing potential, $S$ is the earth surface, $\delta g$ is gravity disturbance, which is the difference between gravity measurement on the earth surface and the normal gravity at the point (it cannot be obtained without knowing ellipsoidal height at the point). $\partial T/\partial h$ represents partial derivative of disturbing potential along the normal plumb line.

Using the spherical approximation, a first-order asymptotic solution of Eq. 1 has the following form (Li et al. 2003, 2005):

$$\zeta_{\mathrm{BVP}} = \frac{R}{4\pi\gamma} \int_{\sigma} (\delta g + \delta g_1) H(\psi) \mathrm{d}\sigma \tag{2}$$

where $\zeta_{\mathrm{BVP}}$ is the global height anomaly, $\gamma$ is the normal gravity on the earth surface, $R$ is the mean radius of the earth, $\sigma$ is the unit sphere, $\psi$ is the angle between the point of interest and integral area element, $H(\psi)$ is the Hotine's kernel (Hotine 1969), $\delta g_1$ is related to ellipsoid height difference between the point of interest and the point of integration, and is given as follows:

$$\delta g_1 = \frac{R^2}{2\pi} \int_{\sigma} \frac{h - h_{\mathrm{P}}}{l^3} \left[ \delta g - \frac{1}{8\pi} \int_{\sigma} \delta g H(\psi) \mathrm{d}\sigma \right] \mathrm{d}\sigma \tag{3}$$

where $l$ is the distance between the integral area and the computing point P and $h - h_{\mathrm{P}}$ is the ellipsoid height difference between the integral area.

Gravity disturbances $\delta g$ can be calculated by (Heiskanen and Moritz 1967)

$$\delta g = g_{\mathrm{P}} - \gamma_{\mathrm{P}} \tag{4}$$

where $g_P$ is the gravity value of observed point P on the ground, and the normal gravity value $\gamma_P$ on each point P can be computed with (Heiskanen and Moritz 1967)

$$\gamma = \gamma_0 + \frac{\partial \gamma}{\partial h} h + \frac{1}{2} \frac{\partial^2 \gamma}{\partial h^2} h^2 + \cdots \tag{5}$$

where $\gamma_0$ is the normal gravity on the reference ellipsoid, and $h$ is ellipsoid height.

From Eqs. 2–5, we can find that the solution of the linearized fixed-gravimetric BVP only requires GPS positions, gravity measurements, and reference ellipsoidal parameters. GPS positions can be easily referenced to the ITRF, which is independent of any local datums. Gravity measurements in different regions can also be tied to an International Absolute Gravity Basestation Network (IAGBN), which is also not related to the local datums. As long as we select the same reference ellipsoid, the (quasi)geoid determined by this method is able to establish a reference surface that contains middle and high frequency components, but without reference to any local datums. Therefore, the height anomalies $\zeta_{\mathrm{BVP}}$ calculated by Eq. 2 is regarded as a "global" height datum.

On the other hand, local height anomalies $\zeta_{\mathrm{local}}$ can be obtained by GPS/leveling data:

$$\zeta_{\mathrm{local}} = h - H^* \tag{6}$$

where $h$ is the ellipsoidal height obtained from GPS and $H^*$ corresponds to the normal height from leveling based on the local vertical datum.

According to Bruns's formula (Heiskanen and Moritz 1967), the following equation can be obtained if common ellipsoidal parameters are adopted for the computation of both local and global height anomalies.

$$\Delta W = \zeta_{\mathrm{local}}\gamma - \zeta_{\mathrm{BVP}}\gamma = (\zeta_{\mathrm{local}} - \zeta_{\mathrm{BVP}})\gamma \tag{7}$$

where $\Delta W$ is the potential difference between the "global" and local height datums. Local height anomalies $\zeta_{\mathrm{local}}$ and the height anomalies $\zeta_{\mathrm{BVP}}$ must correspond to same point on the earth surface and the differences $\Delta\zeta$ can be expressed as

$$\Delta\zeta = \zeta_{\mathrm{local}} - \zeta_{\mathrm{BVP}} \tag{8}$$

When we consider two local height datums, if we calculate their datum potential differences to the "global" datum individually using Eq. 7, the potential difference between two local height datums is given by

$$W_1 - W_2 = \Delta W_{12} = \Delta\zeta_{1A}\gamma_A - \Delta\zeta_{2B}\gamma_B \tag{9}$$

where $W_1$ and $W_2$ represent the gravity potential of the two local height datums, $\Delta W$ is the potential difference between the two local height datums, $\Delta\zeta_{1A}$ and $\Delta\zeta_{2B}$, respectively, represent the height differences between "global" height datum and local height datum at points A and B.

Note that the same set of ellipsoidal parameters should be adopted in the computation, which avoids transformation among different reference ellipsoids.

## 3 Computation and results

### 3.1 Source data

To validate the method discussed in Sect. 2, we use GPS, gravity, and leveling data from Shenzhen and Hong Kong (Fig. 1) to determine the local height datum difference between the two regions. The reasons for the selection of the Shenzhen and Hong Kong regions for the test are as follows:

(1) Shenzhen and Hong Kong local height networks belong to two different datums. Shenzhen height network is referenced to Chinese Height Datum at Qingdao (36°N, 120°E) and Hong Kong network referenced to Hong Kong Principle Datum (22°N, 114°E).
(2) The geographic sizes of both Shenzhen and Hong Kong are relatively small (less than 100 km × 100 km for both regions); therefore, the high frequency components of gravity field cannot be ignored.
(3) As the two regions are close to each other, we can use the geodetic leveling method to evaluate the result using this new method.

Totally 4,870 GPS/gravity points in Shenzhen (with the resolution of 1 km) and 640 GPS/gravity points in Hong Kong (with the resolution from 2 to 4 km) are used for the "global" quasigeoid computation. Totally 37 GPS/leveling points in Shenzhen and 31 GPS/leveling points are used to determine the height anomaly differences between the "global" and local datums.

As the gravity surveys in both regions are connected to the Chinese fundamental gravity network, we can consider that Shenzhen and Hong Kong's gravity systems are the same. GPS networks in Hong Kong and Shenzhen are aligned to the ITRF separately. The Hong Kong GPS network is aligned to ITRF96-1998.121 by GPS network adjustment using one of GPS station in Hong Kong and four other IGS stations in China and Australia. The Shenzhen GPS network is aligned to the ITRF93-1996.365 by connecting GPS stations in Shenzhen to the Chinese fundamental GPS network. Thus, firstly, we have to transform the GPS data from Shenzhen and Hong Kong to the same reference frame ITRF2000 using transformation parameters provided by the IGS. Previous study showed that there is an ellipsoidal height difference of 0.137 m between the Shenzhen and the Hong Kong GPS networks (Luo et al. 2003). The exact reason for this difference is not clear yet (probably the Shenzhen GPS positions were not aligned with ITRF properly, as the GPS data were

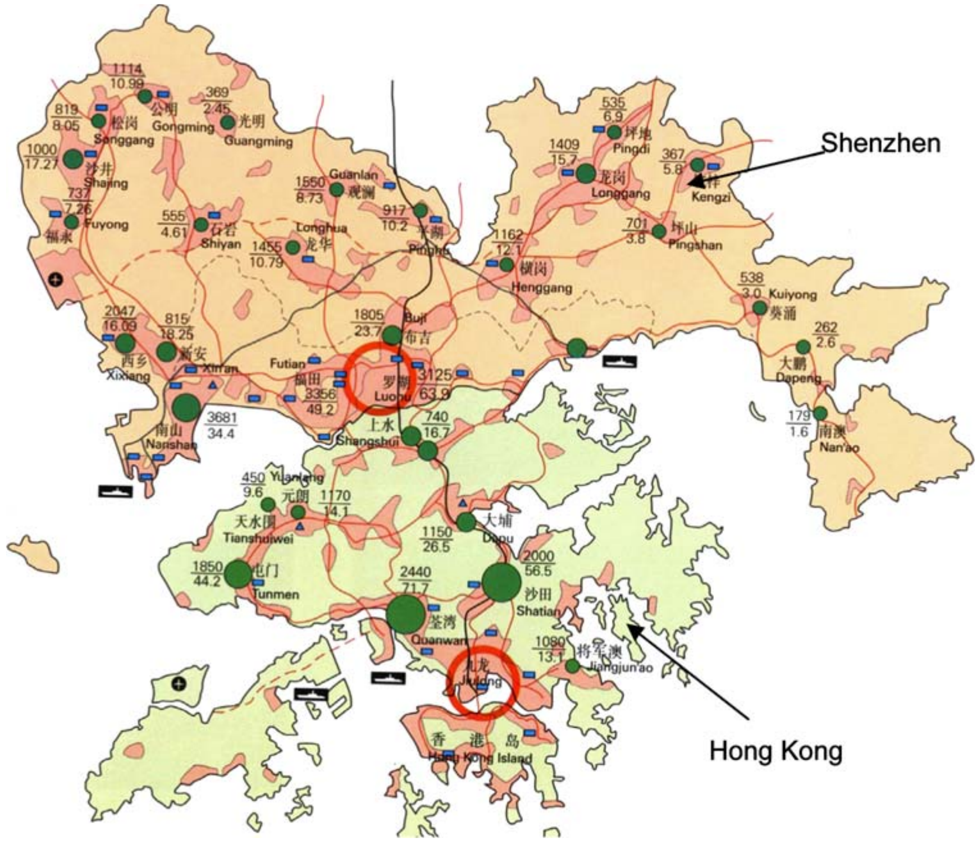


Fig. 1 Geographic locations of Shenzhen and Hong Kong

processed in early 1990s). Therefore, the ellipsoidal height difference is added to all Shenzhen GPS data before the quasigeoid is computed. Both GPS and gravity data are corrected for tidal effects based on the same tidal model (tide-free).

### 3.2 Computation and result

Firstly, we calculate the height anomalies $\zeta_{\mathrm{BVP}}$ for Shenzhen and Hong Kong regions separately using Eq. 2 and the remove-compute-restore technique (e.g. Rapp and Basíc 1992). The EGM96 gravity model is adopted for the computation. The WGS84 ellipsoidal parameters and GPS/gravity data are used to calculate gravity disturbances using Eq. 4. A curve fitting method (Guan et al. 1997) is adopted for grid data computation:

$$\widehat{\delta g} = \begin{cases} \dfrac{\sum\limits_{i=1}^{n} \delta g_i [\varphi(r_i)]^2}{\sum\limits_{i=1}^{n} [\varphi(r_i)]^2} & r_i \neq 0 \\ \delta g_i & r_i = 0 \end{cases} \tag{10}$$

where,

$$\varphi(r_i) = \begin{cases} \frac{1}{r_i} & 0 < r_i \leq \frac{R_0}{3} \\ \frac{27}{4R_0}\left(\frac{r_i}{R_0} - 1\right)^2 & \frac{R_0}{3} < r_i \leq R_0 \\ 0 & r_i > R_0 \end{cases} \tag{11}$$

where $R_0$ is a selected search radius.

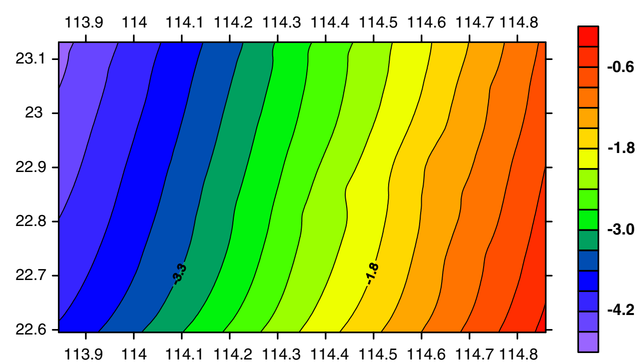


Fig. 2 Height anomaly contour map in Shenzhen. Units: m

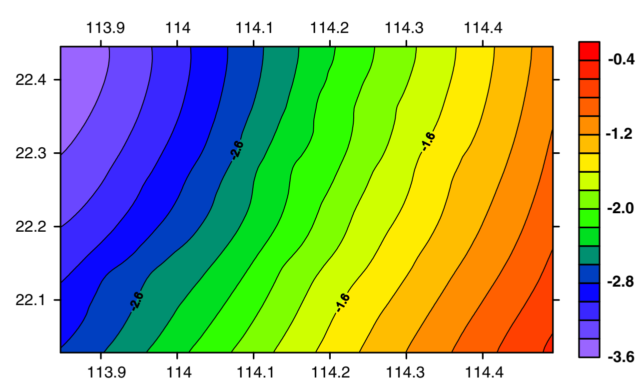


Fig. 3 Height anomaly contour map in Hong Kong. Units: m

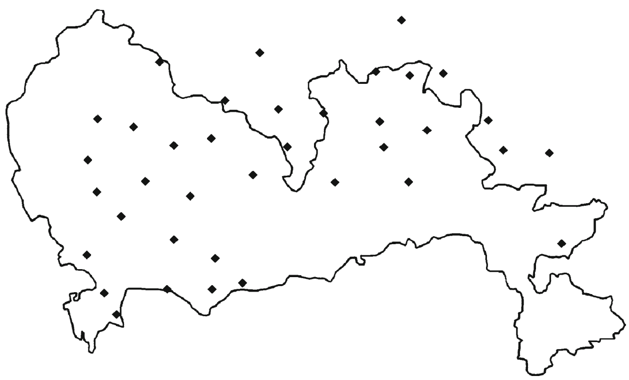

**Fig. 4** The distribution of GPS/leveling points in Shenzhen

**Fig. 5** The distribution of GPS/leveling points in Hong Kong

Using disturbance gravity grid data and Eq. 2, the height anomalies $\zeta_{\mathrm{BVP}}$ can be calculated using a 1D fast Fourier transform (Haagmans et al. 1993). Figures 2 and 3 show height anomaly contour maps in Shenzhen and Hong Kong with the resolution of $1' \times 1'$.

The height anomalies $\zeta_{\mathrm{local}}$ at GPS/leveling points can be calculated using Eq. 6. Figures 4 and 5 show the distributions of GPS/leveling points in Shenzhen and Hong Kong, respectively, which uniformly cover the whole regions. The height differences $\Delta\zeta$ and gravity potential differences $\Delta W$ between height datums can be calculated using Eqs. 7 and 8.

For comparison, we also calculate the height and potential differences referenced to the global EGM96 model (Burša's method). Table 1 shows the statistical results of the comparison between "global" geoids based on the linearized fixed-gravimetric BVP and EGM96 and local geoids based on GPS/leveling points.

From Table 1, we can see that the mean height difference between the Shenzhen and the "global" quasigeoid, based on the linearized fixed-gravimetric BVP, is $0.253 \pm 0.044$ m, and between Hong Kong and the "global" quasigeoid is $-0.644 \pm 0.046$ m. Therefore, the height datum difference between Hong Kong and Shenzhen is $0.897 \pm 0.064$ m. We can also calculate the potential difference between Chinese height datum and Hong Kong datum as $8.780 \pm 0.626\,\mathrm{m}^2\,\mathrm{s}^{-2}$. When we use EGM96 as the global reference surface, the mean height difference between Shenzhen and Hong Kong is $1.151 \pm 0.132$ m. The difference between the two methods is 0.254 m. Also, the standard deviation with the linearized fixed-gravimetric BVP of 0.046 m is about half of that from the EGM96 model of 0.09 m.

In recent years, the Hong Kong and Shenzhen height networks have been connected twice using the geodetic leveling method. The height differences obtained by the two surveys are 0.873 and 0.835 m. Thus, the height difference obtained from the linearized fixed-gravimetric BVP method is consistent with the geodetic leveling results to a few centimeter level.

## 4 Discussion and conclusion

In this paper, we proposed to use the solution of the linearized fixed-gravimetric BVP for the height datum unification in different regions. The main advantage of this method is that the method is not affected by the local height datums, as only gravity and GPS data are required for the quasigeoid computation. As an example, we have estimated the height difference between Hong Kong and Shenzhen height networks. The result is consistent with the geodetic leveling method to a few centimeters.

**Table 1** The difference between Local height anomaly and global height anomaly (units: m)

| | Shenzhen | | Hong Kong | |
|---|---|---|---|---|
| Number of points | 37 | | 31 | |
| Global model | BVP | EGM96 | BVP | EGM96 |
| Maximum difference | 0.350 | 0.354 | −0.566 | −0.815 |
| Minimum difference | 0.172 | 0.022 | −0.722 | −1.175 |
| Mean difference | 0.253 | 0.159 | −0.644 | −0.992 |
| Standard deviation | 0.044 | 0.090 | 0.046 | 0.096 |

Compared with the height unification method based on global gravity field model (i.e. Burša et al. 1999, 2001, 2004), this method does not required data from very large areas, as the high density gravity disturbances contains middle and high frequency gravity information. As Shenzhen and Hong Kong are small regions (less than 100 km × 100 km), the resolution of EGM96 is not sufficient to reflect the high frequency components of gravity field in the regions, and as a result, there exists a large bias of 0.25 m when we use EGM96 model to determine the height difference between Shenzhen and Hong Kong.

To apply this method, GPS positions and gravity survey data in different regions have to be available in the same reference frames. Although the ITRF and Global Absolute Gravity Network (GAGN) have been well established, we need to process the raw data very carefully to maintain them in the same reference frame with sufficient accuracy. The example shows that GPS data from different regions and observed at different times can have significant difference, although all of them claim to have been aligned to the ITRF.

In data processing, we should apply the same tidal correction to both GPS and gravity data. Also, the fundamental geodetic constants and reference ellipsoid have to be consistent in all data processing. Otherwise, these differences may introduce significant biases in height datum unification. Further studies should be carried out for data with different reference frames in different parts of the world.

Today, a main problem for utilizing the proposed method is how to obtain GPS heights at the gravity stations. To resurvey all gravity points with GPS is not realistic, especially not on a global scale or in the region where the gravity points are difficult to be recovered. However, in area where gravity and GPS data can be matched, the proposed method will have obvious advantages. Along with the wide use of GPS, more GPS data will be available. That will improve the situation quickly.

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